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A GRAPHICAL METHOD OF DEDUCING THE CRITERIA FOR THE NATURE OF THE ROOTS OF CUBIC AND QUARTIC EQUATIONS.

BY R. E. GAINES.

LET the cubic equation be:

$$x^3 + 3Hx + G = 0.^* \tag{1}$$

Forming the simultaneous equations:

$$y = x^3, \tag{2}$$
$$y = -3Hx - G, \tag{3}$$

we have two loci the abscissæ of whose intersections are the roots of (1). Now if a tangent to (2) at the point $(x' y')$ be made to pass through $(0, -G)$, its equation $y - y' = 3x'(x^{2} - x')$ easily reduces to:

$$y = 3(G/2)^{\frac{2}{3}}x - G. \tag{4}$$

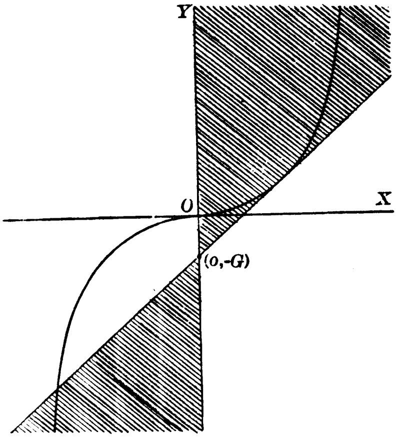

FIG. 1.

Since the right lines (3) and (4) both pass through $(0, -G)$, a comparison of their directions together with the sign of G will give us complete criteria for the nature of the roots of (1). Whether G be positive or negative (see figure 1 as it stands and also turned upside down) one root will be real and the other two will be real and distinct, real and coincident, or imaginary according as (3) lies in the shaded portion of the plane, coincides with the tangent, or lies in the unshaded portion, *i. e.* according as $4H^3 + G^2 <, =, > 0$. Moreover the sign of G will at once determine the signs of the real roots.

The extension of this method to the quartic $x^4 + 6ax^2 + 4bx + c = 0$ is so obvious that the work in detail may be left to the reader. The auxiliary loci are a right line and a quartic. The latter, whose equation is $y = x^4 + 6ax^2$, will assume different forms (see figures 2 and 3) according as a is positive or negative. It will be noticed that not only the origin but also the points M

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* The method we are about to use applies to the general cubic; but for simplicity we take this reduced form.

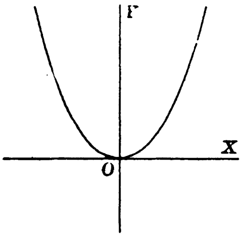

FIG. 2. $a \geqq 0$.

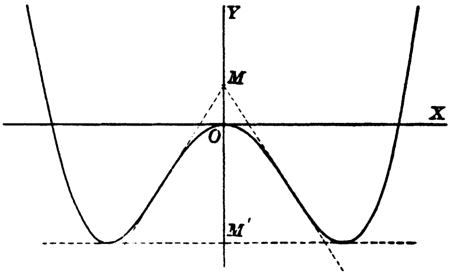

FIG. 3. $a < 0$.

and M', where the Y-axis is met by the tangents at the inflexions and the double tangent respectively, are critical points of this axis.

RICHMOND COLLEGE, VA., DECEMBER, 1899.